**Class XII** | **Sample Question Paper** | Time: 3 hours

**MATHEMATICS** | Max Marks: 80

**GENERAL INSTRUCTIONS:**

1. This question paper contains five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 2 Assertion-Reason based questions of 1 mark each
3. Section B has very short answer questions of 2 marks each.
4. Section C has 6 short answer questions of 3 marks each.
5. Section D has 4 long answer questions of 5 marks each.
6. Section E has 3 source based/case based of assessment (4 marks each) with sub parts.

**SECTION A**

*Multiple Choice Questions (Each question carries 1 mark.)*

1. If R is a relation on Z (set of all integers) defined by x R y iff |x-y| $\leq$ 1, then R is

(a) reflexive and symmetric (b) reflexive and transitive

(c) symmetric and transitive (d) an equivalence relation

2. If R is a relation on the set A = {1, 2, 3} given by R = {(1, 1), (2, 2), (1,3)}, then R is

(a) reflexive (b) symmetric (c) transitive (d) none of these

3. The domain of the function $\cos^{-1}\sqrt{1-x}$ is

(a) [-1.1] (b) $(-\infty, 1]$ (c) [0,1] (d) none of these

4. If A and B are square matrices of same order, then AB'-BA' is a

(a) skew-symmetric matrix (b) symmetric matrix

(c) null matrix (d) unit matrix

5. If A is a 3x3 matrix such that |A| = -2, then $|-2A^{-1}|$ is equal to

(a) 4 (b) -4 (c) 8 (d) -2

6. The function $f(x) = \cot x$ is discontinuous on the set

(a) $\{x = n\pi,\ n \in \mathbf{Z}\}$ (b) $\{x = 2n\pi,\ n \in \mathbf{Z}\}$ (c) $\{x = \frac{n\pi}{2},\ n \in \mathbf{Z}\}$ (d) $\{x = (2n+1)\frac{\pi}{2},\ n \in \mathbf{Z}\}$

7. The number of all possible matrices of order 2 x 3 with each entry 1 or 2 is

(a) 16 (b) 6 (c) 64 (d) 24

8. If $y = e^{\log \sin^{-1} x} + e^{\log \cos^{-1} x}, 0 < x < 1,$ then

(a) $\frac{dy}{dx} = 0$ (b) $\frac{dy}{dx} = \frac{\pi}{2}$ (c) does not exist (d) none of these

9. If $x = t^2$ and $y = t^3$, then $\frac{d^2y}{dx^2}$ is equal to

(a) $\frac{3}{2}$ (b) $\frac{3}{2}t$ (c) $\frac{3}{2t}$ (d) $\frac{3}{4t}$

10. The interval in which the function $f(x) = 2x^3 + 3x^2 - 12x + 1$ is strictly increasing is

(a) $[-2,\ 1]$ (b) $(-\infty, -2] \cup [1, \infty)$ (c) $(-\infty,\ 1]$ (d) $(-\infty, -1] \cup [2, \infty)$

11. A cone whose height is always equal to its diameter is increasing in volume at the rate of $40\ cm^3/sec$. The rate at which its radius is increasing when its circular base area is $1\ m^2$ is

(a) $1\ mm/sec$ (b) $2\ mm/sec$ (c) $0.001\ cm/sec$ (d) $0.002\ cm/sec$

12. $\int \sin\frac{x}{2}\cos\frac{x}{2}\cos x\ dx$ is equal to

(a) $-\frac{1}{4}\cos 2x + C$ (b) $-\frac{1}{8}\cos 2x + C$ (c) $\frac{1}{8}\cos 2x + C$ (d) $-\frac{1}{8}\sin 2x + C$

13. $\int e^{3\log x}(x^4+1)^{-1}dx$ is equal to

(a) $\frac{1}{4}\log(x^4+1) + C$ (b) $-\frac{1}{4}\log(x^4+1) + C$

(c) $\log(x^4+1) + C$ (d) none of these

14. $\int_{a+c}^{b+c} f(x)\ dx$ is equal to

(a) $\int_a^b f(x-c)dx$ (b) $\int_a^b f(x+c)dx$ (c) $\int_a^b f(x)dx$ (d) $\int_{a-c}^{b-c} f(x)dx$

15. $\int \sqrt{1+\sin 2x}\ dx$ is equal to

(a) $-\cos x + \sin x + C$ (b) $\cos 2x + C$ (c) $-2\cos 2x + C$ (d) none of these

16. If $y = Px + \sqrt{a^2P^2 + b^2}$, where $P = \frac{dy}{dx}$. The order and degree of this differential equation is

(a) order = 2, degree = 2 (b) order = 2, degree = 1

(c) order = 1, degree = 2 (d) order = 2, degree = undefined

17. The straight line $\frac{x+2}{5} = \frac{z-3}{-1},\ y = 2$ is

(a) parallel to $x$-axis (b) parallel to $y$-axis (c) parallel to $z$-axis (d) perpendicular to $y$-axis

18. If A and B are two events such that P(A)$\neq 0$ and P(B/A)$= 1$, then

(a) A⊂B (b) B⊂A (c) B= ∅ (d) none to these

***ASSERTION-REASON BASED QUESTIONS***

In the following questions a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices:

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. Assertion (A): Let A be a square matrix of order 2 and adj. (ad). A) = A

Reason (R): |adj. A| - |A|

20. Assertion (A): If a line makes an angle of $\alpha$, $\beta$ and $\gamma$ with the axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$
Reason (R): If a line makes an angle of $\alpha$, $\beta$ and $\gamma$ with the axes, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

**SECTION B**

*(This section comprises of very short answer type questions (VSA) of 2 marks each.)*

21. Solve the equation $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\frac{1}{\sqrt{3}}$

22. Prove that the diagonal elements of a skew-symmetric matrix are all zero.

**OR**

Construct a 3 x 2 matrix whose elements $a_{ij}$ are given by $a_{ij} = \begin{cases} i+j, & if\ i \geq j \\ 2i+j, & if\ i < j \end{cases}$

23. If $y = e^{\tan^{-1}x}$, find $\frac{d^2y}{dx^2}$

**OR**

If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

24. Find the vector whose length is 3 units and which is perpendicular to the vector $\vec{a} = 3\hat{\imath} + \hat{\jmath} - 4\hat{k}$ and $\vec{b} = 6\hat{\imath} + 5\hat{\jmath} - 2\hat{k}$.

25. Two vectors $\hat{\imath} + \hat{k}$ and $3\hat{\imath} - \hat{\jmath} + 4\hat{k}$, represent the two side vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively of triangle ABC. Find the length of the median through A.

**SECTION C**

*(This section comprises of short answer type questions (SA) of 3 marks each.)*

26. Let N denotes the set of all natural numbers and $R$ be the relation on N x N defined by $(a, b)\ R\ (c, d)$ if $ad\ (b + c) = bc\ (a + d)$. Prove that $R$ is an equivalence relation.

27. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ w.r.t. $\cos^{-1}\left(2x\sqrt{1-x^2}\right)$, when $x \neq 0$

**OR**

Differentiate $\sin^{-1}\left[\frac{2^{x+1}.3^x}{1+(36)^x}\right]$ w.r.t. $x$.

28. Find out the least value of $a$ such that the function $f(x) = x^2 + ax + 1$ is increasing on $[1, 2]$

29. Solve the following differential equation: $(x^2 + 3xy + y^2)dx - x^2dy = 0.$

30. Find the shortest distance between the lines whose vector equations are:

$\vec{r} = \left(4\hat{\imath} - \hat{\jmath} + 2\hat{k}\right) + \lambda\left(\hat{\imath} + 2\hat{\jmath} - 3\hat{k}\right)$ and $\vec{r} = \left(2\hat{\imath} + \hat{\jmath} - \hat{k}\right) + \mu(3\hat{\imath} + 2\hat{\jmath} - 4\hat{k})$

31. Three coins are tossed. Consider the events E: ‘three heads or three tails’, F: 'at least two heads', and G: at most two heads. Of the pairs (E, F). (E, G) and (F, G), find which are dependent and which are independent.

**OR**

In a group of 50 scouts in a camp, 30 are well trained in first aid techniques while the remaining are well trained in hospitality but not in first aid. Two scouts are selected at random from the group. Find the probability distribution of number of selected scouts who are well trained in first aid. Also find the mean of the distribution

## SECTION D

*(This section comprises of long answer type questions (LA) of 5 marks each.)*

32. Evaluate: $\int \frac{\sqrt{x^2+1}(\log(x^2+1)-2\log x)}{x^4}\, dx,\ x > 0$

**OR**

Evaluate the following integral: $\int e^{3x} sin\, 4x\, dx.$

33. Find the area of the region included between the curve $4y = 3x^2$ and the line $2y = 3x + 12$.

34. Find the vector and cartesian equations of a line through the point (1, -1, 1) and perpendicular to the lines joining the points (4, 3, 2), (1, -1, 0) and (1, 2, -1), (2, 1, 1).

35. Find the value of $p$ , so that the lines $l_1: \frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2}$ and $l_2: \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other. Also, find the equations of a line passing through the point $(3,\ 2, -4)$ and parallel to line $l_1$.

## SECTION E

*(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1,2 respectively. The third case study question has two sub-parts of 2 marks each.)*

36. Case Study-1 : Read the following passage and answer the questions given below:

In an elliptical sport field, the authority wants to design a rectangular soccer field with the maximum possible area. The sport field is given by the graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(i) If the length and the breadth of the rectangular field be $2x$ and $2y$ respectively, then find the area function in terms of $x$.

(ii) Find the critical point of the function.

(iii) Use First Derivative Test to find the length $2x$ and width $2y$ of the soccer field (in terms of $a$ and $b$) that maximize its area.

**OR**

Use Second Derivative Test to find the length $2x$ and width $2y$ of the soccer field (in terms of $a$ and $b$) that maximize its area.

37. Case Study-2 : Read the following and answer the questions given below:

Consider the following curves $x^2 + y^2 \leq 1$ and $x + y \geq 1$.

On the basis of above information , answer the following questions:

(i) Find points of intersection of both curves.
(ii) Find the common area in sq. units.
(iii) Find the uncommon area of the circle in sq. units.

38. Case Study-2 : Read the following passage and answer the questions given below:

An insurance company believes that people can be divided into two classes : those who are accident prone and those who are not. The company's statistics show that an accident prone person will have an accident at sometime within a fixed one-year period with probability 0.6, whereas this probability is 0.2 for a person who is not accident prone. The company knows that 20 per cent of the population is accident prone.

Based on the given information, answer the following questions:

(i) What is the probability that a new policy holder will have an accident within a year of purchasing a policy ?
(ii) Suppose that a new policy holder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone ?